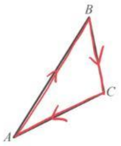
Q1a

In triangle ABC, $\overrightarrow{AB} = 5\mathbf{i} + 8\mathbf{j}$ and $\overrightarrow{BC} = \mathbf{i} - 5\mathbf{j}$



(a) Explain why $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \mathbf{0}$.

zero vector : $0\underline{i} + 0\underline{j}$

or

$\begin{pmatrix}0\\0\end{pmatrix}$

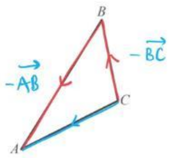
(b) Find $\overrightarrow{CA}$ and calculate its magnitude.

a) $\boxed{\text{The vector sum is a closed path, ending where it starts, so the net movement is zero.}}$

Q1b

Question 1b

In triangle ABC, $\overrightarrow{AB} = 5\mathbf{i} + 8\mathbf{j}$ and $\overrightarrow{BC} = \mathbf{i} - 5\mathbf{j}$



(a) Explain why $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \mathbf{0}$.

(b) Find $\overrightarrow{CA}$ and calculate its magnitude.

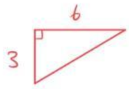
b) $\overrightarrow{CA} = \overrightarrow{CB} + \overrightarrow{BA} = -\overrightarrow{BC} - \overrightarrow{AB}$

$= -(\underline{i} - 5\underline{j}) - (5\underline{i} + 8\underline{j})$

$= \boxed{-6\underline{i} - 3\underline{j}}$

magnitude:

$\sqrt{6^2+3^2} = \boxed{3\sqrt{5}}$



Q2a

a) $\underline{a} + \underline{b} + \underline{c} = \underline{0}$

$\begin{pmatrix}-1\\n\end{pmatrix} + \begin{pmatrix}5\\-4\end{pmatrix} + \begin{pmatrix}m\\6\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix}$

$-1 + 5 + m = 0$ $\quad$ $n - 4 + 6 = 0$

$\boxed{m = -4}$ $\quad$ $\boxed{n = -2}$

Q2b

(a) $\mathbf{a} = \begin{pmatrix} -1 \\ n \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 5 \\ -4 \end{pmatrix}$, $\mathbf{c} = \begin{pmatrix} m \\ 6 \end{pmatrix}$

Given that the resultant of **a**, **b** and **c** is the zero vector, find the values of m and n. [2]

(b) $\mathbf{d} = \begin{pmatrix} -3k \\ k \end{pmatrix}$

$\underline{i}$ component

$\underline{j}$ component

Given that $|\mathbf{d}| = 2\sqrt{15}$, find two possible values for k. Give your answer as an exact value.

b) $|\underline{d}| = \sqrt{(3k)^2 + k^2} = 2\sqrt{15}$

$9k^2 + k^2 = 4(15) = 60$

$k^2 = 6$

$\boxed{k = \pm\sqrt{6}}$

Q3

$A(3k, -17k)$

$(-17k) = (3k)^2 - 2$

$9k^2 + 17k - 2 = 0$

$(9k - 1)(k + 2) = 0$

$k = \frac{1}{9}, \cancel{-2}$ ← reject since k must be positive

sub $k = \frac{1}{9}$ into coordinates for A.

$x = 3\left(\frac{1}{9}\right) = \frac{1}{3}$ $\quad y = -17\left(\frac{1}{9}\right) = \frac{-17}{9}$

$\boxed{A\left(\frac{1}{3}, -\frac{17}{9}\right)}$

Q4

$\underline{a} - \underline{b} = \begin{pmatrix} 3 \\ 5 \end{pmatrix} - \begin{pmatrix} -3k \\ k \end{pmatrix}$ $\quad\quad$ $\underline{a} + \underline{c} = \begin{pmatrix} 3 \\ 5 \end{pmatrix} + \begin{pmatrix} 0 \\ -4 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} 3 + 3k \\ 5 - k \end{pmatrix}$

For the vectors to be parallel, one must be a SCALAR MULTIPLE of the other.

$$\begin{pmatrix} 3+3k \\ 5-k \end{pmatrix} = p \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

$3 + 3k = 3p$ ① $\quad\quad$ $5 - k = p$ ②

Sub ② into ① to eliminate p.

$3 + 3k = 3(5 - k)$

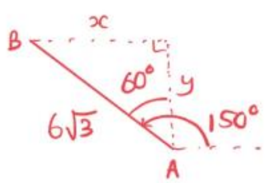
$= 15 - 3k$

$6k = 12$

$\boxed{k = 2}$

Q5a

i)



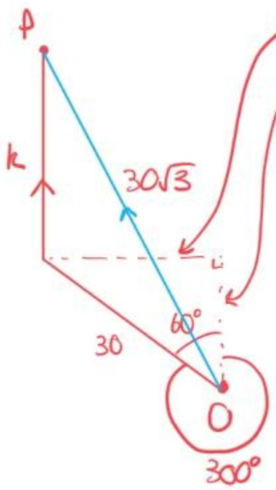
$x = 6\sqrt{3}\sin 60 = \boxed{9}$

$y = 6\sqrt{3}\cos 60 = \boxed{3\sqrt{3}}$

Q5b

b) $\dfrac{\overrightarrow{AB}}{|\overrightarrow{AB}|} = \dfrac{9\underline{i} + 3\sqrt{3}\underline{j}}{6\sqrt{3}} = \boxed{\frac{\sqrt{3}}{2}\underline{i} + \frac{1}{2}\underline{j}}$

Q6



$30\sin 60 = 15\sqrt{3}$

$30\cos 60 = 15$

$\overrightarrow{OP} = 15\sqrt{3}\underline{i} + (15+k)\underline{j}$

$|\overrightarrow{OP}| = \sqrt{(15\sqrt{3})^2 + (15+k)^2} = 30\sqrt{3}$

$(15\sqrt{3})^2 + (15+k)^2 = 2700$

$675 + (15+k)(15+k) = 2700$

$225 + 30k + k^2 = 2025$

$k^2 + 30k - 1800 = 0$

$(k+60)(k-30) = 0$

$k = \cancel{-60}, 30$

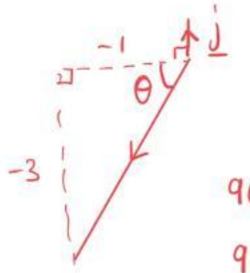
$x = 15\sqrt{3}$

$y = 15 + 30 = 45$

$\boxed{x = 15\sqrt{3} \quad y = 45}$

Q7a

a) The angle between $\underline{R}$ and $\underline{j}$ is the same as the angle between $(-\underline{i} - 3\underline{j})$ and $\underline{j}$ since $\underline{R}$ and $(-\underline{i} - 3\underline{j})$ are parallel.



$90 + \theta$

$90 + \tan^{-1}\left(\frac{3}{1}\right) = \boxed{161.57^\circ \text{ (2dp) anticlockwise}}$

Q7b

b) $\underline{R} = (5+x)\underline{i} + (y-3)\underline{j}$ $\qquad (-\underline{i} - 3\underline{j})$

Since these two vectors are parallel, the ratios of their $\underline{i}$ and $\underline{j}$ components must be equal!

$$\frac{5+x}{y-3} = \frac{-1}{-3}$$

$$-15 - 3x = 3 - y$$

$$\boxed{3x - y = -18}$$

Q7c

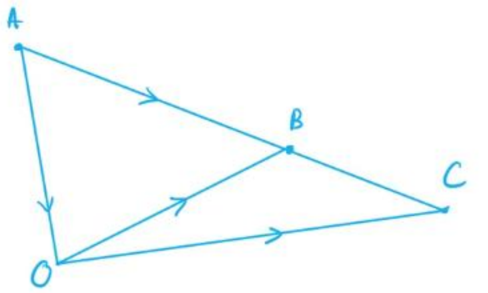
c) When $y = -3$

$$3x - (-3) = -18$$
$$x = -7$$

$$\underline{R} = (5-7)\underline{i} + (-3-3)\underline{j}$$
$$= -2\underline{i} - 6\underline{j}$$

$$|\underline{R}| = \sqrt{2^2 + 6^2} = \boxed{2\sqrt{10}}$$

Q8



scalar quantity

To be parallel: $\overrightarrow{AB} = p\ \overrightarrow{AC}$

$$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = -\overrightarrow{OA} + \overrightarrow{OB} = -(-9\underline{i} + 4\underline{j}) + -6\underline{i}$$
$$= 3\underline{i} - 4\underline{j}$$

$$\overrightarrow{AC} = -\overrightarrow{OA} + \overrightarrow{OC} = -(-9\underline{i} + 4\underline{j}) + (3\underline{i} - 12\underline{j})$$
$$= 12\underline{i} - 16\underline{j}$$

×4! ($p = 4$)

Since $\overrightarrow{AC} = 4\overrightarrow{AB}$, the two vectors are parallel, and since A is a common point, they must lie on a straight line (they are collinear).

Q9A

a) G lies on $\overrightarrow{BC}$, so $\overrightarrow{BG}$ is parallel to $\overrightarrow{BC}$, and therefore $\overrightarrow{BG}$ is a scalar multiple (λ) of $\overrightarrow{BC}$. $\overrightarrow{BG}$ is shorter than, or equal to $\overrightarrow{BC}$, but acts in the same direction, therefore $0 \leq \lambda \leq 1$.

Q9B

b) $\overrightarrow{FG} = \overrightarrow{FB} + \overrightarrow{BG}$

$\overrightarrow{BG} = \lambda \overrightarrow{BC}$

$= \frac{n}{m+n}\underline{a} + \lambda \overrightarrow{BC}$

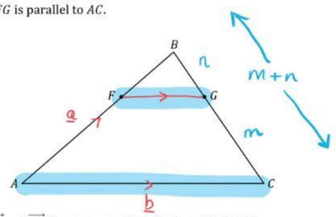
$\overrightarrow{BC} = -\underline{a} + \underline{b}$

$= \frac{n}{m+n}\underline{a} + \lambda(-\underline{a} + \underline{b})$

$$\boxed{\overrightarrow{FG} = \left(\frac{n}{m+n} - \lambda\right)\underline{a} + \lambda \underline{b}}$$

Q9C

In triangle ABC, point F lies on AB and point G lies on BC.
F divides AB in the ratio $m:n$.
The line segment FG is parallel to AC.



(a) Explain why $\overrightarrow{BG} = \lambda\overrightarrow{BC}$ for some constant λ, where $0 \leq \lambda \leq 1$.

[1

(b) Given that $\overrightarrow{AB} = \mathbf{a}$ and $\overrightarrow{AC} = \mathbf{b}$, show that

$$\overrightarrow{FG} = \left(\underbrace{\frac{n}{m+n} - \lambda}_{=0}\right)\mathbf{a} + \lambda\mathbf{b}$$

[4

c) $\frac{n}{m+n} - \lambda = 0$

$\overrightarrow{FG}$ is parallel to $\underline{b}$, so the component in the direction of $\underline{a}$ must be zero!

$\frac{n}{m+n} = \lambda$

$\overrightarrow{BG} = \lambda \overrightarrow{BC} \longleftarrow$ from (a)

$\overrightarrow{BG} = \frac{n}{m+n}\overrightarrow{BC}$

$$\boxed{\begin{array}{c}\overrightarrow{BG} : \overrightarrow{GC} \\ n : m\end{array}}$$